

Unity in Divine Complexity: *A Paracomplete Constructivist Model*

RAMON HARVEY

Cambridge Muslim College

rh@cambridgemuslimcollege.ac.uk

Abstract: I argue that a focus on articulating God's unity in terms of divine simplicity has overshadowed the exploration of alternative theories of divine complexity. Assessing the compatibility between models of the divine nature and formulations of unity, I claim that, whereas the principle of unity implicitly upheld within common models of divine simplicity begs the question against complexity, the rival notion of inseparable unity is insufficient. I therefore propose a stricter non-question-begging principle of unity that is only applicable to divine complexity within a paracomplete constructivist logical framework; that is, one rejecting the inviolability of the Law of Excluded Middle (LEM) and allowing for a logical underdetermination, or gap, in the case of God. In the remaining part of the article, I develop my model in greater detail, discussing its logical underpinnings; application to the categories of entity and property within a constituent ontology; and use of constructive set theory to account for multiple intensional properties. Finally, I show how my model allows for the logically consistent articulation of strict unity for a complex divine nature.

Keywords: Constructivist, Paracomplete, Divine simplicity, Divine complexity, Unity, Intensionality

Introduction

Monotheists typically do not just hold that there is one God, but that the sole divine being is a perfect unity. How ought such a concept of unity be understood? One of the most common ways that theologians and philosophers of religion have conceived of unity is through the doctrine of divine simplicity, explaining it in terms of complete undifferentiation. Versions of this theory have had defenders in Christianity, Islam, Judaism and other traditions, retaining support

up to today (Vallicella 2023; Brower 2009; Zarepour 2022).¹ Yet there is also a long history of unity being upheld within models of divine complexity, even if not always in name; in fact, this is the majority view in Islam (Arslan 2024). Despite some important recent contributions (O'Connor 2023; Baddorf 2017), there are many aspects of the question of divine complexity that have not been addressed in contemporary philosophical theology. Hence, this article aims to take stock of the current debate and to advance it by proposing a novel model that leverages paracomplete constructive logic.

There are numerous possible motivations for holding a stance of divine complexity. It is often taken to be desirable scripturally and rationally to affirm distinct attributes or properties for God. That is, many theists take the apparent language of scripture to imply God possesses distinct properties: qualities such as knowledge and power.² Moreover, it can seem puzzling how God can know and create the world without them,³ notwithstanding the counterarguments available to the supporter of simplicity (Brower 2009; Dolezal 2011). In any case, the general question of the viability of divine complexity is not my focus here, but rather its compatibility with unity.

To enter the debate over unity in divine complexity is to revisit in some form the problem of the One and the Many, a venerable philosophical puzzle that goes back to at least Parmenides.⁴ The problem can be said to have two aspects: (i) In what sense does a given theory of divine complexity affirm multiple properties? And (ii) what principle unifies these multiple properties, such that God is a singular unified entity?

With respect to the first aspect, I will discuss two general models for divine complexity: 'substratum' theories that understand God in terms of a bare particular that possesses additional properties and 'bundle' theories that negate

¹ In Christianity, there is both a debate over divine unity in light of the multiple properties that God may seem to possess, and the so-called logical problem of the Trinity, which deals with the reconciliation of unity with the three divine 'Persons'. See Hasker (2013); Branson (2019). This latter question will not be addressed here. In Islam, the discussion over the central notion of God's oneness (*tawhīd*) is focused on how multiple divine properties, or predicates, are to be explained according to divine unity. See El-Bizri (2008). God's unity is a foundational principle in Judaism, especially in the influential formulation of Maimonides. See Hyman (1967, 132). Also relevant in this context is the Neoplatonic concept of 'the One', which is principally understood in terms of unity (Wildberg 2021).

² In the Bible, see for example 1 Chronicles 29:11, Psalms 62:10–11, Proverbs 2:3–5, Isaiah 11:9, Jeremiah 10:12, Matthew 22:29, Romans, 11:33. In the Qur'an, see 2:165, 2:255, 4:166, 11:14, 35:11.

³ Consider the implication of the following plausible principle: 'Power-knowledge difference (PKD): For any being B with power and knowledge, the truths of the propositions 'B is powerful' and 'B is knowledgeable' are grounded in non-identical parts of B' (Baddorf 2017, 408).

⁴ For discussion of Plato's account of the Parmenidean version of the problem, see Allen (1974).

the former element.⁵ Standard versions of these theories identify unity with the inseparability of metaphysical constituents. I will propose that this can be considered a too permissive definition of unity and that it is possible to suggest an alternative one, a principle of logical unity that can only be met by a model of divine complexity when it utilises paracomplete constructive logic – specifically a logical underdetermination, or gap. This leads me to my main contribution in this article: the articulation of my own model of divine complexity that is consistent with this stricter approach to unity.

I will claim that for God to have strict unity in complexity, the following three conditions must be met without logical contradiction: (i) to possess multiple properties that are distinct from one another; (ii) to not be identical to these properties; (iii) to not be distinct from these properties. In such a model, God has multiple mutually distinct properties, but these properties are neither sufficient to account for His identity, nor is there anything extra that needs to be accounted for. This leaves a logical gap in which God's unity is expressed by a lack of determination in the relationship to His properties. In other words, God is an entity that defies the Law of Excluded Middle (LEM) or what has sometimes been called in the literature an 'odd object' (see below). This means I shall use a paracomplete constructive logic that allows for genuine underdetermination – gaps between identity and distinctness – while rejecting true contradictions. Although the divine case is the focus of the present article, I do not rule out other instances of underdetermination applied to ideal or factual objects.

In order to articulate a model that meets the above three conditions, I will work from a region of logical space that upholds a paracomplete constructivist approach to logic and set theory.⁶ My focus is on how the intensional profile of an entity can be understood by the way that its set of properties map to it. This

⁵ For this article, I take a model of God as providing 'a particular coherent set of unique claims about the nature of God and the God-world relation' (Mullins 2022), though I only focus on the first part. A validly constructed model is valuable because it lets one know what can, logically speaking, truly correspond to factual reality. The distinction between such logical space and the factual world is often related to the debate over the existence of abstract objects. This can be unhelpfully framed as a choice between 'platonism', which accepts their existence, and 'nominalism', which rejects it (both are to be distinguished from the historical views of the same names); see Falguera et al. (2022). The problem to my mind is that the dichotomy between the real existence of such objects held by the platonist and the non-existence or reduction to concrete objects held by the nominalist obscures much of what is interesting about the types of objects in question. See Husserl (1975, 25). I will work on the assumption that the two realms can be adequately distinguished for the purpose of the model that I will develop without seeking to settle their precise status or relationship here.

⁶ I understand 'constructivist' here in two related senses. In general terms, it refers to the fact that logical space is defined in terms of ideal constructions – it has an epistemic not an ontic status. More specifically, it refers to the demand that one works with constructible propositions, ones that require positive evidence, which in some contexts disallows use of LEM.

leads me to propose a novel Axiom of Intensionality to define in general the intensional profiles of entities in terms of their properties. The axiom allows for an explication of paracomplete logical derivations to express unity in complexity for God.

The structure of the article is as follows. In Section 1, I take stock of the debate over the complexity of the divine nature. In Section 2, I discuss principles of unity and articulate the one that I will use in my constructive proposal. Section 3 covers preliminaries relating to the background paracomplete logic. In Section 4, I apply these logical notions to a constituent ontology consisting of the categories of entity and property. Section 5 explores multiple properties, bringing in constructive set theory to account for intensional properties. Finally, in Section 6, I show how God's unity can be upheld despite multiple distinct properties by defying LEM.

1. Divine Complexity

I take divine complexity to refer to the denial of divine simplicity, which amounts to the affirmation that God has metaphysical constituents.⁷ In the theistic context in which I am writing, these metaphysical constituents are to be understood as properties or attributes, and not corporeal parts.⁸ The usual understanding of the metaphysical work that such properties do is to serve as the ontological ground for various true predications about God. For example, the predications 'God is knowing' and 'God is powerful' are made true by mutually distinct features of the divine nature: God's knowledge and power. This model requires a constituent ontology, rather than a relational one (since on a relational ontology any entity lacks metaphysical constituents),⁹ meaning at the very least that an

⁷ It is possible to argue that, more precisely, divine complexity affirms that God has at least one metaphysical constituent that is non-identical to Himself, since divine simplicity can accept such constituents so long as each is identical to Himself. For reflection on this point, see Oppy, 2003. Brower frames it as follows: 'If an intrinsic predication of the form "God is F" is true, then *God's F-ness* exists and is identical with God'. Brower (2009, 108). This, of course, only follows if the idea of a self-identical constituent is coherent. No part of my argument turns on this distinction, but it does affect the phrasing of certain points. I will write so as to retain neutrality over whether a simple God lacks any metaphysical constituents or merely lacks non-self-identical constituents. What is important for my argument is that a metaphysical constituent is not inherently distinct (in my paracomplete sense) from its possessing entity.

⁸ In the literature, the term 'metaphysical part' is quite common, though I will try to avoid it as much as possible.

⁹ Divine properties within a relational ontology would be abstract objects exemplified by God (Crisp 2019, 565) and, according to van Inwagen, 'if properties, like propositions and proper relations, are abstract objects, there is no possible sense of 'constituent' in which a property can be a constituent of an individual...' (van Inwagen 2011, 52). Whether or not van Inwagen is right on this point, I hold the divine properties meant by the term 'metaphysical constituents' to be

entity has its properties as constituents (Koons & Pickavance 2017, 179). This acknowledges that it is possible to differentiate between these two fundamental types in an asymmetrical fashion. God possesses multiple properties, whereas each of those properties can only be possessed by Him.¹⁰

The key feature of such a model of divine complexity, then, is the possession of metaphysical constituents by God who is understood as an entity or concrete particular. The main candidates for metaphysical constituents are distinct properties, and if asserted, a bare particular or, as I will term it, a substratum that possesses them. Hence, according to commonly held models, divine properties must belong to God and are what can be called ‘inseparable’ or ‘essential’, such that it is impossible that they could be removed or fail to belong to Him.¹¹ Despite some arguments to the contrary, this can be defended as successfully preserving divine aseity (Baddorf 2017; Steele and Williams 2019). Moreover, complexity in this aspect of the divine nature does not prevent simplicity in other ones, for instance in the essence and existence distinction (Baddorf 2017). Hence, at least for the present article, God will be taken as identical to His essential nature, or essence, and to His existence in the sense that He is the sole factual counterpart to the ideally distinct statements of *what* He is and *that* He is.¹²

There are two main models for God’s nature that can be proposed as rivals to divine simplicity: (i) Substratum theory and (ii) Bundle theory.

- (i) Substratum theory: God is a substratum with essential properties that are distinct from the substratum and each other.
- (ii) Bundle theory: God consists of essential properties that are distinct from each other.

Substratum theory posits that God is understood as a metaphysically complex entity in which a substratum (sometimes referred to as the Godhead, or less

concrete (not abstract) and factual (not ideal). The relational model seems to me to raise other problems related to divine unity, but I will not pursue them here.

¹⁰ This way of stating the problem avoids the debate over universals, which I will not engage here and need not engage, since God can be understood as a particular with unique properties.

¹¹ I focus on such core properties as knowledge and power since it is broadly agreed that God should be essentially omniscient and omnipotent. There is a discussion about whether any of God’s properties should be understood as inessential, which I will not address here. See O’Connor, 2023. In any case, the model for unity in complexity that I will go on to articulate does not affirm such inessential properties.

¹² In other words, the truthmaker for ‘God’s nature is x where x are essential properties’ and ‘God exists’ is in both cases God, such that ‘essence’ and ‘existence’ are conceptually and not factually distinct. This is not, then, a point that must be uniquely upheld by proponents of divine simplicity. Cf. Platter (2021, 11–12).

helpfully as God's 'essence') possesses metaphysically distinct inseparable properties. According to this theory, the divine nature has two kinds of constituents, a single substratum that acts as the bearer of properties, and the mutually distinct properties that are so borne.¹³ In some contexts, only the former is referred to as God, for instance in the sentence: 'God possesses multiple distinct properties'. Yet as these properties are essential for God, the ontological model that describes the complete divine reality includes both the substratum and its properties.

Bundle theory does away with the substratum since God is understood as a metaphysically complex entity consisting of mutually distinct essential properties. In other words, the bundle theorist does not think that a substratum is needed to possess or to be ascribed the properties, but rather the entity, God, possesses them as His metaphysical constituents. On this view, then, God is a single entity understood through these properties.¹⁴ With these models in place, I shall discuss the question of divine unity.

2. Divine Unity

In this section I will bring a challenge to bear on the aforementioned models of divine complexity based on the concept of unity. This, then, requires an exploration of what meaning ought to be given to this concept. Some philosophers and theologians take for granted the idea that any form of complexity for the divine nature cannot be unified. They might implicitly posit something like the following principle:

- (i) Principle of Simple Unity: An entity has essential unity if and only if any metaphysical constituents are identical to itself.

The idea would be that any kind of internal distinction for an entity, which is required for it to be complex, brings with it a corresponding lack of essential unity. It seems that a given constituent that is non-identical to the entity could have been logically (even if not metaphysically) absent from it, thereby threatening its unity. In other words, the claim is that only a simple entity can be

¹³ For an example of this model in Duns Scotus, see Steele and Williams (2019); for Scotus and Palamas, see Spencer (2017). Islamic versions are surveyed in Andani (2024). Whereas the natural reading of Scotus' model is as a theory of complexity, Spencer includes it within an expanded 'flexible' notion of simplicity. Though my view is that the obvious dividing line to draw for the term simplicity is the negation of all non-identical metaphysical constituents, I am interested here in discussing the implications of the underlying models for the notion of divine unity, rather than adjudicating on the use of the name, past or present.

¹⁴ An example of this model is Michael Istvan's reading of Spinoza. See Istvan (2020).

truly unified, since it lacks any internal metaphysical distinctions and could not logically be in any way different than it is (Crisp 2019, 562). Though the Principle of Simple Unity is a viable position to hold in and of itself, it begs the question for divine simplicity within the present dialectic by defining unity – the concept under investigation – in terms of simplicity.

An alternative view, clearly at play in the two formulations of divine complexity that I have already articulated, is to consider unity consistent with distinct constituents so long as they are metaphysically necessary. The principle in this case would be as follows:

- (ii) Principle of Inseparable Unity: An entity has essential unity if and only if it lacks inessential metaphysical constituents.

This principle would appear to avoid the above question-begging assumption, since it allows both for the unity of divine simplicity, which upholds an entity that certainly lacks inessential metaphysical constituents (if it is construed as having a self-identical constituent in the first place), as well as the two models of divine complexity above. Yet there is a worry that such a principle is too permissive on the question of unity.¹⁵ There would seem to be an aspect of unity in complexity that is not captured by the models already discussed. In both cases, it is indeed necessary for God to possess His given metaphysical constituents. But there is no principle of unity beyond that. Mere compresence of constituent elements seems insufficient to establish the special singularity that is held as appropriate for God.

I do not deny that the essentiality of metaphysical constituents satisfies an important modal aspect of unity, but query whether it answers the ‘counting’ problem that emerges from the problem of the One and the Many: when we say there is *one* God, what are we counting? In the case of Substratum theory, the obvious candidate for the one count is the single substrate (i.e. the Godhead) with properties as concomitant necessary features. The problem on that reading is that *what is one* does not exhaust the complete divine reality. On Bundle theory, there

¹⁵ One might be minded to argue that, on the contrary, this principle is too strict. This could be first because it rules out the essential unity of bodily objects in the world that possess inessential, i.e. accidental, metaphysical constituents. Though I have not specified a physical ontology here, I would see it is an advantage for any entity with inessential metaphysical constituents (i.e. presumably any worldly thing) to not have the same essential unity status as God. This does not preclude the entity having some weaker form of unity: e.g. unity in terms of causal coherence within the world or, from the ideal standpoint, mere possession of an intensional profile (see below). Second, one might want to consider certain properties, such as ‘creating the world’ for a free God, as inessential. I think that this inessentiality can be avoided, whether that is by treating the relevant properties as essential or parsing them as features of the world, not God. In any case, these points are downstream of the current inquiry.

is no substrate, so the one count is to the collection of properties grasped together. Here the worry is reversed: strictly speaking, there is no additional factual entity *that is one* beyond the many properties. The Principle of Logical Unity that I will explore in this article addresses both problems. By ruling out metaphysical constituents that are distinct from God, *what is one* both exhausts the totality of God and is the factual entity *that is one*, rather than a mere collection. It acts, therefore, as a sufficiency principle to supplement necessary inseparability.

I propose the following definition:

- (iii) Principle of Logical Unity: An entity has essential unity if and only if it lacks metaphysical constituents that are distinct from it.¹⁶

First, this definition does not beg the question of whether an entity (God) has metaphysical constituents. If God is simple, then He lacks any metaphysical constituents (or at least any distinct ones) and so fulfils the definition. Yet it is inclusive of a model in which God possesses non-identical metaphysical constituents so long as they are not distinct from Him. With this in mind, let us consider the two basic versions of divine complexity that I have already introduced. As already established, on Substratum theory, God (construed as the substratum or Godhead) is distinct from the divine properties, whereas on Bundle theory God is distinct from any one of the individual properties that make up the bundle.¹⁷ From this, both theories fail this principle.

The manner in which the Principle of Logical Unity has been framed permits a restricted class of divine complexity models. If God is not to be simple, then He must possess metaphysical constituents, yet these metaphysical constituents must not be distinct from Him. On the classical assumption that identity and distinctness are exclusive and fully exhaustive, whatever is not distinct from something is identical to it. But identity to purported metaphysical constituents is usually considered a version of divine simplicity. Thus, it would seem this principle would lead to simplicity as the only way for God to possess unity by collapsing back into the Principle of Simple Unity.

I argue, therefore, that for God with multiple metaphysical constituents to be unified according to the Principle of Logical Unity, one must not apply LEM to the question of whether God's metaphysical constituents are either identical to

¹⁶ Throughout this article, I take distinctness as a primitive constructive predicate, which is incompatible with identity but not reducible to non-identity. See Section 3 below. It should also be clear that the negation of distinctness cannot be construed as metaphysical inseparability, which would collapse the Principle of Logical Unity into the Principle of Inseparable Unity.

¹⁷ One might ask: what would happen if the bundle consists of a single property? In that case, God would be simple and identical to that property. Something along these lines is commonly held by proponents of divine simplicity for whom God is identical to His act.

or distinct from Him. That is, by taking a paracomplete constructivist approach, as I elaborate below, there is no exhaustiveness to this disjunction. Rather, a third undetermined ‘gap’ status can be affirmed in answer to this question. This idea may raise some eyebrows. I think it is important to head off two objections at this juncture for those who are either worried that a disavowal of LEM for God would collapse bivalence throughout logical activity or that this is just a very strange and implausible claim to make in the first place. In response to the former point, as I will explore in more detail below, what I mean by the disavowal of LEM is to reject it as an inviolable logical law in all syntactic contexts, not as a reliable logical norm in wide areas of thought. Regarding the latter point, I would reply that it should be expected for God to be a special case. Philosophers and theologians have variously suggested, for example, that speaking about God is impossible, requires paraconsistency, or must be done via negation. In this context, discussing divine unity via paracompleteness should not be seen as singularly bizarre, but rather taken as another option within the menu of solutions to this puzzling question. In order to illustrate the viability of this idea in the remainder of this article, I will develop a specific model for it in some detail.

3. Logical Preliminaries

In order to articulate a model of God, it is necessary to affirm certain logical rules and by doing so take up a stance in relation to logical space.¹⁸ By specifying that space, one locates oneself within the wider logical universe. This location makes a difference to the propositions that can be validly expressed with our formal tools.

I affirm the Law of Identity (LI), such that $\neg(y = y)$: ‘it is not the case that y is identical to y ’ is taken to be necessarily false. I shall also take the Law of Non-Contradiction (LNC) as self-evident. Consider $(y = x) \wedge (y \neq x)$: ‘ y is identical to x and y is distinct from x ’. This sentence is a logical glut or contradiction. By the semantics I assume below, such a sentence must be evaluated as false according to LNC. To allow a true contradiction would lead to a variety of paraconsistent logic, which is a strategy that I will not pursue (see Priest et al., 2022). Instead, I propose a paracomplete constructivist approach that does not treat LEM as an inviolable logical law. This means that for some formulas A , neither A nor $\neg A$ is

¹⁸ The term ‘logical space’ is often taken by philosophers to refer to the modal notion of all ‘possible worlds’. I have a broader conception in mind, akin to what István Aranyosi defines as ‘a space of possible regions obtained constructively by way of logical operations on propositions’ (Aranyosi 2013, 9). The important features of this notion of logical space are as follows: it allows for all possible logics and mathematics that can be expressed as categorical judgements; and it is an ideal, abstract realm expressible through a formal symbolic language. See Husserl (2001, 165–66). For further relevant discussion about the scope of logical space, see Ahsan (2025).

derivable. Semantically, this can be understood as a constructive three-valued semantics, with propositions evaluated as true, false and (in some cases) ‘undetermined’.

In what follows, I take $=$ as a primitive identity predicate. I also treat $\neq$ as a primitive distinctness predicate, which is incompatible with identity on pain of contradiction, yet not equivalent to negated identity. This reflects my constructive framework: a failure of identity $\neg(y = x)$ does not establish distinctness ($y \neq x$), because distinctness is a positive claim requiring its own evidence. Hence, I argue that the disjunction $(y = x) \vee (y \neq x)$: ‘ y is identical to x or y is distinct from x ’ does not hold universally and in some special cases can fail (Carnielli and Rodrigues 2020, 570). The typical motivation for this move is to delimit logical space to only allow mathematical propositions that are constructible, with the most important aspect being the disavowal of LEM in the above sense (Dummett 2000, 8–12). Ross Brady argues that the basic subject matter of logic is consequence – proof of conclusions from premises – rather than propositions that one may suppose exhaustively capture truth. Hence, such derivations of proof are liable to underdetermination, as it is only when every question has a yes-or-no answer that this possibility is avoided, and this is usually not available (Brady 2019, 163–65). On the other hand, conceptual overdetermination leads to contradiction. It is in this way that the assumption of LEM as a general rule supports the formulation of paradoxes. Under LEM, the Liar paradox ‘This statement is false’, must either be affirmed (L) or negated ($\neg L$). Yet the result of either of the two exhaustive options is overdetermination (because if it is true, it is false, and vice versa), and hence a contradiction ensues (Brady 2019, 167–68).

Another argument against taking LEM as a logical law comes from reflecting on the way that its neutral syntactic role is undermined by exceptions based on a failure of relevance. Consider the following two sentences: ‘A geometric square is prime’ and ‘A geometric square is not prime’. It may look like they are governed by LEM, such that negation of one entails affirmation of the other. Yet these two sentences can either be evaluated as false or left unevaluated, provoking the need for a third value, since the lack of a relevant determinate is explained by a geometric square, unlike an integer, not being the kind of thing that can be prime. This, then, is an example of a sentence with syntactic but not semantic content. Hence, while it is grammatically comprehensible, it seems to escape the remit of LEM (Husserl 1977, 220–21).¹⁹ I understand the phenomenon of well-formed

¹⁹ Related to this are statements that are problematic due to their lack of material reference. For instance, Bertrand Russell’s famous pair of statements ‘The present king of France is bald’ and ‘The present king of France is not bald’ may also be said to both be evaluable as false because there is no present king of France. Yet, unlike a prime or non-prime geometric square, there could be a king of France with or without hair. For analysis of this case, see Sainsbury (2010, 81–82).

syntactic propositions lacking semantic determinates to be a key example of the underdetermination that a paracomplete logic can represent by allowing the failure of specific $A \vee \neg A$ sentences. This analysis raises the problem that if LEM is akin to LI or LNC, then it should be a self-evident logical law that operates on the syntactic level to lead from premises to proven conclusions. As such, its truth should not depend on the semantic content to which it applies (McKenna 1982, 21–22). If it does so depend, then it starts to look like a generally reliable logical norm and not an invariable logical law.

To develop my argument, then, I will stipulate a failure of the disjunction $(y = x) \vee (y \neq x)$: ‘ y is identical to x or y is distinct from x ’ for the ‘odd object’ case (ultimately, God), by assuming $\neg((y = x) \vee (y \neq x))$.²⁰ In the background paracomplete logic I am using, De Morgan’s laws hold,²¹ so this is equivalent to:

$$(1a) \neg(y = x) \wedge \neg(y \neq x): \text{‘}y \text{ is neither identical to } x \text{ nor distinct from } x\text{’}.$$

For ease of expression, I will henceforth express this with the following notation:

$$(1b) y \approx x: \text{‘}y \text{ is neither identical to } x \text{ nor distinct from } x\text{’}.$$

Though expressing a logical gap, this formulation is entirely consistent with LNC. This reflects the basic independence of LEM from this law. That is, the fact that a logical glut is taken as false does nothing to motivate the falsity of a logical gap.²² This is what allows for the possibility of a three-value paracomplete logic.

²⁰ As pointed out by a reviewer, in standard intuitionistic logic, while $A \vee \neg A$ is not derivable for every A , the double negation $\neg\neg(A \vee \neg A)$ is provable for every A . This means that actively asserting $\neg(A \vee \neg A)$ for some case is inconsistent. If my formula is read as $\neg((y = x) \vee \neg(y = x))$, then the general provability of $\neg\neg((y = x) \vee \neg(y = x))$ would derive a contradiction. This does not apply in my case, however, because I treat $\neq$ constructively as a primitive, such that $y \neq x$ is not equivalent to $\neg(y = x)$.

²¹ I do not need to adopt a fully axiomatised paracomplete logic for the present article. Rather, it suffices to work against a background in which LI, LNC and De Morgan’s laws hold, but distinctness is primitive and LEM is not taken as a general logical law. In ordinary cases, there is no consequence from my position that the latter does not universally hold. It only becomes relevant when dealing with specified undetermined situations, such as the identity of ‘odd objects’.

²² Basing logic on the consequence relation, LNC can be expressed in terms of the truth of $A \wedge \neg A \models B$ (proposition A and not- A entails any B), which is the principle of explosion that anything follows from a true contradiction. Conversely LEM can be expressed as the truth of $A \models B \vee \neg B$ (any A entails either B or not- B), which is that bivalence of truth follows from any true statement. Though LNC is widely considered more intuitive than LEM, the above entailments show why these ‘classical’ laws of thought can be adopted independently of each other (Restall, 2004, pp. 75–76). This differs from the premodern view of Avicenna, for example, who argued that a violation of LEM entails a violation of LNC (Zolghadr 2020, 36).

The gap between identity and distinctness expressed in formula (1a)/(1b) will be the basic kernel for the arguments to follow about God. So far, however, the key variables y and x lack any semantic content. To move beyond a purely logical treatment to one that has an application as a metaphysical model, I will need to introduce a framework for discussing the formula in relation to factual reality.

4. Entity and Property

My objective is to derive general formulae for the unity of an entity in terms of its properties, so that I can show how my model of God meets the three conditions stated above. This requires interpreting variables within formulae in such a way as to model the two ontological concepts of entity and property, which I am purposefully keeping as sparse as possible. Although my intention here is not to defend a specific metaphysical theory of properties, I require a minimal ontological language to develop my argument at all. As I have already argued, the kind of entity-property unity that I am pursuing aligns with a constituent ontology. With this idea in place, I can use y to stand for an entity and x for a certain property that is its constituent. Hence, formula (1b) can be interpreted as follows:

(1c) $y \approx x$: ‘entity y is neither identical to property x nor distinct from it’.

The use of this logical gap makes y into what has been called an ‘odd object’ (Goswick 2021, 1629). An ‘odd object’ in this context is one in which certain properties are undetermined. In the present formula, there is a determinate in play with respect to the relationship of entity y to property x : it could have been defined as identical to or distinct from x . But there is no determinate for that relationship. The obvious conclusion is that the lack of a determinate is due to the ‘odd object’ genuinely violating LEM; in other words, it is metaphysically undetermined and not just epistemically unknown. That is, within the paracomplete logic, both the claim that the entity is identical to the property and the claim that it is distinct from it fail to be true, leaving it in an undetermined status.

One may want to resist this conclusion, however, by proposing that there might be a way to explain away the object’s lack of a determinate without affirming its metaphysical status as undetermined. To put the idea another way: an entity modelled according to formula (1c) violates LEM by definition. Yet it might be supposed that the resulting ‘odd object’ after the formula’s interpretation cannot be LEM-defying in the real world. An example of this approach is the ‘negated disjunct strategy’, which insists that in the case of an object lacking determinates, one can negate both disjuncts without this leading

to a LEM violation for corresponding factual objects (Goswick 2021, 1626–27).²³ This seems to be a variant of sentences that fail due to material reference, leading to an explanation from background metaphysical facts.²⁴ For example, in the sentence: ‘So-and-so was either inside or outside the room’, we can appreciate that both disjuncts could be negated without that requiring an ‘odd object’ status for the person in question – perhaps they were standing in the doorway. For the present case, this would mean the negation of both disjuncts of $\neg(y = x) \wedge \neg(y \neq x)$: ‘entity y is neither identical to property x nor distinct from it’ without affecting the assumed bivalent metaphysical dichotomy between identity and non-identity.

I think this strategy is undermined by my sparse ontology here, which shows it is not a principled objection to my model. In this case, there are no metaphysical facts in question other than the identity/distinctness status of the relation between the entity y and property x (I introduce multiple properties x_i below). This cannot be a failure of reference, because entity and property are the proper relata for the identity question to be raised.²⁵ Hence, I shall hold that formula (1c), and the other formulae to follow in this article, consistently model a factually coherent LEM-defying entity. In order to get closer to a model of divine complexity, however, the above formula will first need to be extended for dealing with entities possessing multiple properties. This will require the careful integration of constructive set theory.

5. Multiple Properties and Intensionality in Constructive Set Theory

The principles of logic can be extended to set theory, which allows for the modelling of metaphysical structures. This is useful for the proposed model of divine complexity, in which God will have multiple properties and will need to be defined in contrast to other potential entities with multiple properties. Underlying parallels between the elements of sets (or classes) and the properties of entities make it possible to use with modification the formal framework of set theory: ‘The predication relation between properties and objects corresponds to

²³ Notice that this applies some of the logical strategies for dealing with LEM-defying statements in Section 3 to entities.

²⁴ See note 19.

²⁵ One might object that even the minimal positing of the property-entity distinction brings with it the knowledge that a property (or properties) cannot be identical to an entity in the first place. But this seems obviously question-begging and is contradicted by theories of divine simplicity that affirm this idea. In Section 5, I define an entity intensionally in terms of its properties in such a way to leave open a range of relationships, which correspond to different possible metaphysical scenarios. In any case, for the purposes of my argument, what matters is that within a paracomplete constructivist framework there is room to consistently model an LEM-defying entity.

the membership relation between sets and elements: Px is analogous to $x \in C'$ (Shapiro 2002, 17). Adoption of set theory, including constructive set theory, in the context of questions of entity and property identity may suggest application of the Axiom of Extensionality:

$\forall A \forall B (\forall x (x \in A \Leftrightarrow x \in B) \Rightarrow A = B)$: 'For all sets A and B , A is identical to B if and only if they have the same elements x '.

But this axiom falls short in dealing with properties, which are intensional. That is, whereas two sets with identical extensions have the same members and so by the axiom are the same set, two ideal descriptions of entities with multiple properties need not be. To give an example, the set of 'existing things with a volume defined by $4/3\pi r^3$ ' and 'existing things with a surface area defined by $4\pi r^2$ ' both have the same extension, a certain number of spheres in the world. But these two expressions have different intensions, reflecting two different properties (see Bealer, 1998). In fact, it is plausible that property ascription is not just an instance of intensionality but its paradigm case (Shani 2007, 164–65). In what follows, I will explore the intensional definition of entities in terms of the particular contribution that all their properties make to their identity. I refer to this concept as an intensional profile.

To take account of the intensionality of properties, I will introduce the idea of mapping an entity's properties to a function that defines the relation between the entity and those properties. In Zermelo-Fraenkel set theory (ZF), a function maps extensional elements from one set to another. Within my constructive framework, I will map between properties and related constructions, such as intensional profiles. Though I retain set theoretic notation for its familiarity and accessibility, the kind of mapping that I use within formal language allows for direct modelling of the kind of intensional relationships between properties and their entities that are found within the factual world rather than that within abstract sets.

I propose an Axiom of Intensionality, which explains the way that the intensional properties of entities constitute their intensional profiles:²⁶

Axiom of Intensionality: $\forall y_1 \forall y_2 (I(y_1) = I(y_2) \Leftrightarrow \forall xi (R(y_1, xi) \leftrightarrow R(y_2, xi)))$:
'For any pair of entities y_1 and y_2 , the intensional profile of y_1 is identical

²⁶ It is surprising that such an axiom for properties has not been discussed more in the philosophical literature. Leaving aside strictly mathematical treatments, one of the main lines of development for reflection on the idea seems to have been twentieth-century responses to Bertrand Russell's *Introduction to Mathematical Philosophy*. For example, Kurt Gödel informally discussed such an axiom for use with concepts as the principle 'that to different definitions belong different notions'. (Gödel 1984, 457–58).

to the intensional profile of y_2 if and only if for all properties xi , the properties xi of entity y_1 have the same relational mapping as the properties xi of entity y_2' .²⁷

The principle at work here is that only when one can relationally map the complete set of properties from one entity to another in the same way (and vice versa) is it proven that the two entities have an identical intensional profile. This amounts to an intensional variation of Leibniz's Identity of Indiscernibles.²⁸ It should be noted that intensional identity corresponds to the more familiar idea of an essential nature or essence. Whether that equates extensionally to a single or to multiple factual entities is not decided by intensional identity alone and would require wider metaphysical argumentation. It may be provisionally suggested, however, that multiple realisations of the same intensional profile would have to be on non-essential grounds, such as spatiotemporal location or accidental properties. I think, therefore, that there are good prospects for further establishing God's factual existence and uniqueness, though it is not the objective of the present study.

The Axiom of Intensionality provides a suitable context to introduce a formula to define the intensional profile of an entity within the language of constructive set theory:

- (2) $I(y) = \{xi \mapsto R(y, xi) \mid xi \in y\}$: 'The intensional profile of an entity y is identical to the mapping of its properties xi to the relational function between y and xi (in which xi are constituent properties of y)'.

Formula (2) expresses the contribution made by each of the entity's properties to its identity in the context of the relation of those properties to the entity. $I(y)$ is the entity's essential nature or essence, since by the Axiom of Intensionality an alternative mapping of intensional properties would equate to a distinct entity. This is a flexible framework that can accommodate a range of metaphysical scenarios about the entities so modelled. For instance, if $R(y, xi)$ expresses $y = xi$, this could be interpreted as a variety of bundle theory in which the entity in question is identical to the set of its properties. On the other hand, if $R(y, xi)$ expresses $y \neq xi$, this could refer to a substratum theory in which the entity is not identical to its properties, since they are understood as additional to the substratum. Neither of these options, which correspond to the standard models of divine complexity discussed above, will be explored further in the present context, since my aim is to model a complex entity, God, that fulfils the Principle

²⁷ Here $\forall xi$ ranges over properties that are constituents of at least one of the compared entities (e.g. $xi \in y_1$ or $xi \in y_2$).

²⁸ See the similar observation in Gödel (1984, 457, n. 17).

of Logical Unity and so cannot be expressed within the classical logical binary of equalities.²⁹

6. Divine Unity through Undetermined Properties

To recap, I have presented a general formula (2) for the intensional understanding of an entity y within the selected region of logical space. An entity's intensional profile is defined by the mapping of its properties to the relational function between itself and its properties. With this in place, I will now return to formula (1c), my starting point for an LEM-defying entity in terms of its property identity, and modify it for multiple properties. In the case of the model of divine complexity, the logical gap expressed by formula (1c) provides the relational function needed. Hence, I derive:

- (3) $R(y, xi) = y \approx xi$: 'the relational function between entity y and properties xi is that y is neither identical to properties xi nor distinct from them'.

Substituting $R(y, xi)$ into formula (2):

- (4) $I(y) = \{xi \mapsto y \approx xi \mid xi \in y\}$: 'The intensional profile of an entity y is identical to the mapping of its properties xi to ' y is neither identical to xi nor distinct from xi '.

This expression further extends the unity of the mapping of properties xi to the intensional profile of y . Whereas the relational function in (2) makes clear that the properties of any entity play an integral role in its intensional profile, (3) expresses God's special relationship to His properties as neither identical to nor distinct from them. Formula (4) incorporates this singularity into a specific intensional formula for God's relationship to His properties. It is impossible on this formulation to assert that He is intensionally either a mere bundle of individual properties (according to the first disjunct) or that He is a substratum possessing them as additional elements (according to the second disjunct). Rather God's identity-distinctness status is evaluated as undetermined. This model, then, meets the condition of the Principle of Logical Unity, such that God has essential unity because he lacks metaphysical constituents that are distinct from Him. Moreover, it does not force one to deny the distinctness of one property from another, since the properties have been defined from the outset as mutually

²⁹ By defining the intensional profile of an object in terms of the mapping of its properties to the relational function between the entity and its properties, the Axiom of Intensionality and its derivations allow for a range of relational functions, such that this function can remain determinate in the case of worldly objects or undetermined in the case of God.

distinct.³⁰ The significance of this formulation is that God is affirmed in a logically consistent manner to be constituted by all His properties without the composition that would follow were the relational function to be $y = xi$ or on the assumption of a substratum on $y \neq xi$. In this carefully defined paracomplete constructivist way, God is understood as a singular unity constituted by His properties without dissolving the distinctness between them.

Conclusion

I have argued that models of divine simplicity typically define unity in ways that presuppose undifferentiation, whereas standard models of divine complexity involve an insufficient concept of unity based on the inseparable compresence of properties. I have shown that a stricter non-question-begging Principle of Logical Unity is not only met by divine simplicity but also by divine complexity on a background paracomplete constructive logic. To develop my own model, I proposed an Axiom of Intensionality from which I have constructed a series of formulae to express the unity of God with His properties. I have argued that God has multiple intensional properties, which are mutually distinct. Yet His intensional profile is defined as a unity through the mapping of those properties to the relational function that He is neither identical to nor distinct from them – a logical gap that returns a value of undetermined to the question of identity-distinctness status. The consistency of this model does not, of course, imply that the ideal description of God in the above framework corresponds to the existence of God in the factual world. The question of the factual existence, or extension, of the entity intensionally defined here would require a different set of metaphysical arguments, ones that I have not raised. Nevertheless, apart from its inherent logical interest, this model contributes a new perspective to a longstanding debate in the philosophy of religion over God's consistent possession of multiple attributes while remaining a unity.

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³⁰ In other words, asserting $y \approx xi$ does not entail $x_1 \approx x_2$, since the constructive approach allows for gaps between objects in domains without forcing gaps between objects in subdomains.

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